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CS-225: Discrete Structures in CS

Homework 3, Part 2

Instruction: Use the method of proof by contraposition or/and by contradiction **only** if proving the problem statements given below -

Exercise Set 4.7 of the required textbook:  Question  #13, #24, #29  
Exercise Set 4.8  of the required textbook:  Question  #7,  #18 (a)

13. Let S be the statement: “The product of any irrational number and any nonzero rational number is irrational.”

∀ irrational number(r) and nonzero rational number(s), the product is irrational

a. Write a negation of S: There exists an irrational number and a nonzero rational number whose product is rational.

b. Prove S by contradiction: Proof: Suppose not. That is, suppose there is an irrational number and a nonzero rational number whose product is rational.

Since the integer c above is not defined as nonzero, the denominator has the potential to equal zero due to the rules of multiplication. With the rules of division, a zero denominator cannot equal a rational number. Thus shows rs≠ rational.

24. The reciprocal of any irrational number is irrational. (the reciprocal of any nonzero real number x is 1/x)

Formal: ∀ irrational number x, 1/x is irrational.

1. Contraposition: Suppose you have any fraction with 1 as the numerator and any rational number (a) as the denominator. [We must show that a is rational]
   1. ¼ = .25 (rational due to a definitive end to the decimal)
   2. a=4 which is a rational number. Therefore a is rational by definition of rational.
2. Contradiction: Suppose there exists an irrational number (x) whose nonzero reciprocal (1/x) is rational.

Therefore an irrational number whose nonzero reciprocal cannot be rational due to the definition of irrational and the counterexample [as was shown].

29. For all integers m and n, if m+n is even, then m and n are both even or m and n are both odd.

Contradiction: Suppose not. Suppose that for some integers m and n, if m+n is even then m is even and n is odd, or n is even and m is odd. m+n produces an even number s. By definition of even and odd, m=2k and n=2k+1 and s=2k., for some integer k

by basic algebra

Since k does not equal an integer by definition, then m cannot be even when n is odd in the formula m+n [as was to be shown above]

7.

Suppose not. Suppose that is rational. Then by definition of rational:

by adding 7 to each side

By the rule for adding fractions

by adding fractions with a common denominator and dividing each side by 3.

Then a+7b and 3b are integers by the laws of integers. 3b ≠ 0 by the zero product property. Hence √ 2 is a quotient of two integers (a+7b and 3b, where 3b≠ 0). By the definition of rational this makes √ 2 rational which contradicts the fact that √ 2is rational. Hence

18. a. Prove that for every integer a, if is even then a is even. [This statement is true]

Suppose not. Suppose that there is an integer ‘a’ such that if is even then a is odd.

An odd integer: a=2k+1

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By definition of odd, a is an odd number [as was to be shown]. Hence the initial statement is true.